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AN EXTENDED ALGORITHM FOR THE OCEAN TIDE TERM IN THE FORCE MODE--ETC(U)
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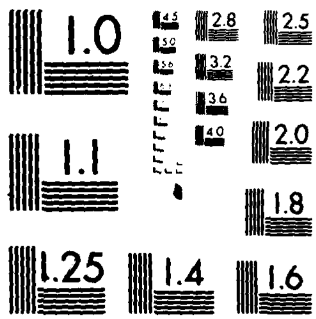

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A revised and extended version is presented of a computer algorithm specifying the gravitational action by which the ocean tides perturb satellite motion.

The current ocean tide term in the CELEST and TERRA equations of motion is based entirely on the semidiurnal (M_2) tide mode. Also, the corresponding solid earth tide term neglects ocean loading. The present algorithm for the perturbing acceleration caused by the ocean tide re-formulates

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the M_2 tide and introduces ten additional spectral components (S_2, N_2, K_2, K_1, O_1, P_1, Q_1, M_f, M_m, S_{sa}). In addition, the effect on the solid earth of hydrostatic loading by the ocean tide is now included.

FOREWORD

Several computer algorithms were recently developed at the Naval Surface Weapons Center (NSWC), accounting for the gravitational action that the tidal dislocations of the earth's masses exert upon artificial satellites. The necessary physics background and the computer routines themselves were detailed in five technical reports between 1976 and 1979. Included in that body of work were a theory of and a computer program formulation for the perturbing acceleration caused by the ocean tide. This was based entirely on the semidiurnal (M_2) tide constituent. Also, the corresponding sclid earth tide term in the equations of motion ignored ocean loading.

Since then, an effort was made to re-formulate the ocean tide, introducing the S_2, N_2, K_2, K_1, O_1, P_1, Q_1, M_f, M_m, S_{sa} spectral constituents. In addition, the effect on the solid earth tide of bottom loading by the ocean tide was included, for convenience, in the perturbing acceleration due to the ocean tide, removing the need for an extensive modification of the solid earth tide term.

The work documented here was done in the Space and Surface Systems Division in support of computer program development for satellite geodesy. Joseph M. Futcher, Jr. of the Physical Sciences Software Branch is Programmer. The author is further obliged to Robert A. Manrique of the SLBM Research and Analysis Division for contributing computer results that verified the author's checkout calculations.

This report was reviewed by R. L. Kulp, Head, Space and Ocean Geodesy Branch; D. R. Brown, Jr., Head, Space and Surface Systems Division; and R. J. Anderle, Research Associate of the Strategic Systems Department.

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CONTENTS

INTRODUCTION

Work was recently undertaken to establish theories and computer algorithms for the gravitational action by which the tidal redistributions of the earth's masses perturb satellite orbits. From this effort resulted five technical reports[1-5] of which the first four document the physics background and details of the tide models. The fifth specifies the computer routines for the various perturbing accelerations resulting from these models, thus permitting the tide effects to be included into the equations of motion of the CELEST and TERRA computer programs.

Contained among the tidal perturbing terms was an algorithm (Chapter 3, Pages 17 through 31, of Reference 5) for the ocean tide. This started from an existing table for the global tide amplitudes and phase angles[6-8], postulating that each tidal height value resembles a gravitating point mass that by its presence perturbs the satellite motion. Only the semidiurnal (M_2) lunar tide was considered. But this algorithm was regarded as setting the pattern for introducing other components of the fluid tide later. The restriction has since been removed because amplitude and phase angle tables have become available for the S_2, N_2, K_2, K_1, O_1, P_1, Q_1, M_f, M_m and S_{sa} constituents of the ocean tide. It then became possible to revise the existing computer program for the perturbing acceleration due to the M_2 tide by formulating the perturbing terms for the just mentioned additional members of the sea tide spectrum.

Part of the 1979 collection of tide algorithms[5] was a computer program formulating the Newtonian attraction caused by the tide bulge of the solid earth. This included both the lunar and the solar tide components. The tidal properties were assumed to be functions of latitude. Accordingly, Love coefficients were featured that are zonal expansions of latitude. An allowance was made for tide lag manifesting itself as a time delay in the response of the tidal mass redistribution to the tidal stress field. Absent was however the effect of ocean tide loading on the tide bulge. It appeared soon desirable to account for ocean tide loading. To avoid having to burden the algorithm for the land tide with the very numerous parameters specifying the ocean tide, it was decided (and found possible) to leave the former unchanged and to reflect the necessary alterations in the computer routine for the latter. This happened to be not only the most economic but also, from the viewpoint of the tide physics involved, the most natural way of doing things. The task turned out to consist merely of a trivial modification of the tidal point mass in the ocean tide model.

REVISED COMPUTER ALGORITHM FOR THE PERTURBING ACCELERATION CAUSED BY THE OCEAN TIDE INCLUDING THE OCEAN LOADING EFFECT ON THE SOLID EARTH TIDE ROUTINE

INPUT DATA

Concerned are the ocean tide modes M_2, S_2, N_2, K_2, K_1, O_1, P_1, Q_1, M_f, M_m and S_{sa}. For each of these tides, there exists a computer tape that lists the NP values of the tidal amplitude ζ_{ij} and the tidal phase δ_{ij}.

NP Number of grid points. This may be expected to be a five-digit integer.

ζ_{ij} Tidal amplitude on area element ij. If available in meters, use directly. Otherwise, convert to meters.

δ_{ij} Tidal phase angle on area element ij, in degrees.

Custodian of the tapes just mentioned is Ling Szeto of the Physical Sciences Software Branch. Also required are

R "Radius of the Earth," i.e., the semimajor axis of a suitable reference ellipsoid, in kilometers. Trial value: $R = 6378.145\ km$.

ε^2 Square of eccentricity of reference ellipsoid. Trial value: $\varepsilon^2 = 0.00669342$. In case it is desired to start from the ellipsoid flattening, f, find ε^2 from $\varepsilon^2 = (2 - f)f$.

μ_E Gravitational constant of the earth, in kilometers and seconds. Trial value: $\mu_E = 398601\ km^3\ sec^{-2}$.

G Newton's constant, in *km*, *kg* and *sec*.
Trial value: $G = 6.6732E\text{-}20\ km^3\ kg^{-1}\ sec^{-2}$.

ϱ Density of water, in *kg* and *km*.
Trial value: $\varrho = 1.E + 12\ kg\ km^{-3}$.

ϱ^* Density of the ocean bottom, in *kg* and *km*.
Trial value: $\varrho^* = 3.E + 12\ kg\ km^{-3}$.

σ Rate of mean mean longitude of moon.
$$\sigma = \frac{180}{\pi}\ 1.40519E\text{-}04\ deg\ sec^{-1}.$$

The following parameters J, n and t^* speci . the time instant at which the perturbing acceleration due to the tide is to be found. Normally, this is the time instant associated with the current time line of the orbit integration.

J Number of the calendar year.

n Number of the day within the year.

t^* Mean solar time at Greenwich (GMT, UTC), in seconds.

$\left.\begin{array}{r} x_1 \\ x_2 \\ x_3 \end{array}\right\}$ Inertial, Cartesian components of satellite position vector in *km*, associated with the just specified time instant. For the precise definition of the reference frame, see TERRA and CELEST documentation[9].

$(ABCD)_{J,n,t^*}$ Transformation matrix that manages the transition from the inertial frame of the satellite equations of motion ("Basic Inertial System," which is associated with either 1950.0 or with the beginning of the day UT of the trajectory epoch) to the earth-fixed reference frame corresponding to the time instant J, n, t^*. This transformation is a computer routine that is already part of TERRA and CELEST. For details, see the TERRA and CELEST documentation[9].

$NMAX$ Limit on the number of terms in the expansion of the tide potential.

COMPUTER ROUTINE FOR THE EXPANSION COEFFICIENTS OF THE TIDE POTENTIAL

The time-independent constituents of the expansion coefficients (${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, ${}^{\beta}H_{nm}$) of the tide potential will now be calculated from the amplitude and phase angle computer tapes, separately for each of the eleven ocean tide modes.

Once established, the coefficients ${}^{\alpha}F_{nm}$, ${}^{\beta}F_{nm}$, ${}^{\alpha}H_{nm}$, ${}^{\beta}H_{nm}$ will remain unchanged as long as the amplitude and phase tape for the particular tide mode remains valid. Thus, the computer routine under discussion will be exercised quite infrequently, namely just once each time a new edition of the just mentioned amplitude and phase tape becomes available. That is expected to occur at intervals of several years, during which the present routine will remain dormant.

For each ocean tide mode execute the following algorithm. For the geometry associated with the index ij, see Figures 1, 2 and 3.

$$\alpha_{ij} = 10^{-3}(\varrho - 0.0667\varrho^*)\, G\, \Delta S_{ij}\, \zeta_{ij} \cos \delta_{ij} \tag{01}$$

$$\beta_{ij} = 10^{-3}(\varrho - 0.0667\varrho^*)\, G\, \Delta S_{ij}\, \zeta_{ij} \sin \delta_{ij} \tag{02}$$

$$\Delta S_{ij} = \left(\frac{\pi}{180}\right)^2 R^2 \sin\left(\frac{\pi}{180} j\right) \tag{03}$$

= area of the non-polar surface area element for $j = 2, 3, 4, \cdots, j_{MAX} < 180$

$$(\Delta S_{ij})_{POLAR} = \frac{1}{2}\left(\frac{\pi}{180}\right)^3 R^2 \tag{04}$$

= area of the polar surface area element (neighboring the North Pole)

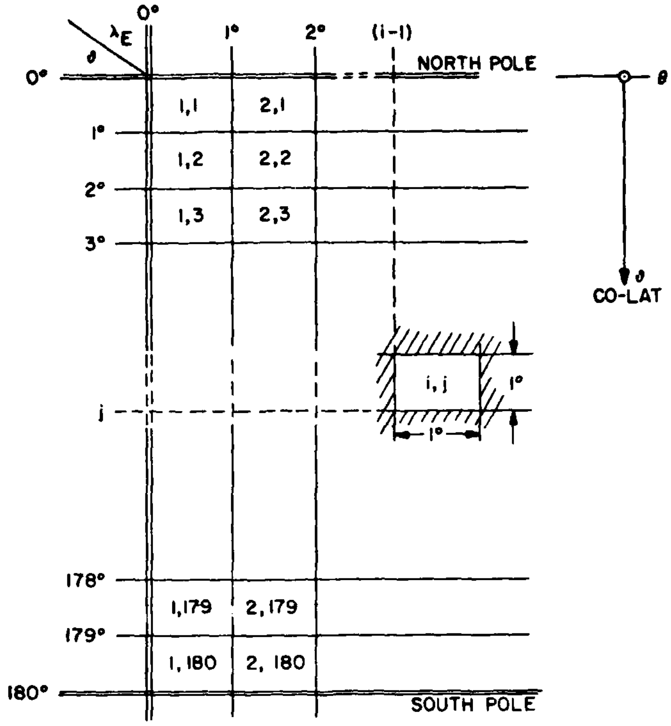

Figure 1. Divison of the Earth's Surface Into Area Elements According to the Schwiderski Ocean Tide Model

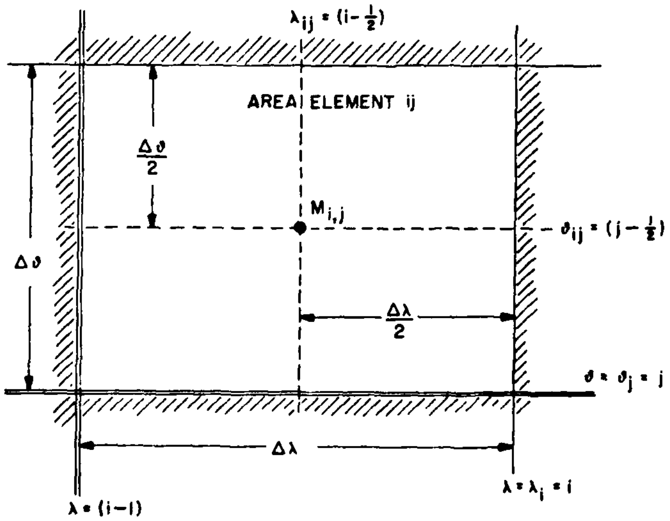

Figure 2. Position of Point Mass at the Geometrical Center of the Surface Area Element Associated With *ij*

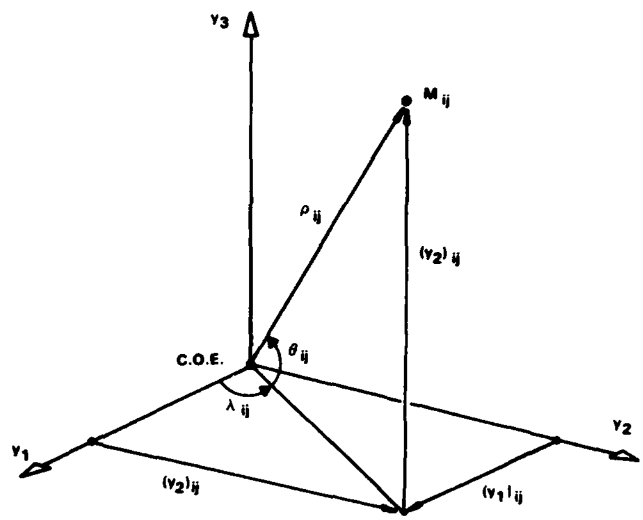

Figure 3. Position of the Point Mass M_{ij} in the Earth-Fixed Cartesian Coordinate Frame

$$\theta_{ij} = \frac{\pi}{2} - \frac{\pi}{180}\left(j - \frac{1}{2}\right) \text{ in radians} \tag{05}$$

$$\lambda_{ij} = \frac{\pi}{180}\left(i - \frac{1}{2}\right) \text{ in radians} \tag{06}$$

$$\varrho_{ij} = \left(1 - \frac{\varepsilon^2}{2}\sin^2\theta_{ij}\right) \tag{07}$$

To each index ij assign now a counting number ν, $1 \leqslant \nu \leqslant NP$.

$${}^{\alpha}F_{nm} = \left(2 - \delta_m^0\right) \frac{(n-m)!}{(n+m)!} \frac{1}{\mu_E R^{2n}} \sum_{\nu=1}^{NP} \varrho_\nu^{2n+1} \alpha_\nu f_{nm}^\nu \tag{08}$$

$${}^{\beta}F_{nm} = \left(2 - \delta_m^0\right) \frac{(n-m)!}{(n+m)!} \frac{1}{\mu_E R^{2n}} \sum_{\nu=1}^{NP} \varrho_\nu^{2n+1} \beta_\nu f_{nm}^\nu \tag{09}$$

$${}^{\alpha}H_{nm} = \frac{(n-m)!}{(n+m)!} \frac{1}{\mu_E R^{2n}} \sum_{\nu=1}^{NP} \varrho_\nu^{2n+1} \alpha_\nu h_{nm}^\nu \tag{10}$$

$${}^{\beta}H_{nm} = \frac{(n-m)!}{(n+m)!} \frac{1}{\mu_E R^{2n}} \sum_{\nu=1}^{NP} \varrho_\nu^{2n+1} \beta_\nu h_{nm}^\nu \tag{11}$$

$$\delta_l^k = \begin{matrix} 1 \\ 0 \end{matrix} \; if \; \begin{matrix} l = k \\ l \neq k \end{matrix} \tag{12}$$

Calculate the f^v_{nm} and h^v_{nm} as follows, noting that

$$p_v = \frac{R}{\varrho_v} \tag{13}$$

and

$$h^v_{no} = 0 \text{ for all values of } n \tag{14}$$

and that

$$h^v_{nm} = f^v_{nm} = 0 \text{ for any } n < m. \tag{15}$$

Obtain, separately for each v, the required f^v_{nm} and h^v_{nm} from the following recurrence relations:

To advance in n, evaluate

$$f^v_{n+1,m} = \frac{p_v}{n - m + 1} [(2n + 1) \sin \theta_v f^v_{n,m} - (n + m) p_v f^v_{n,m}] \tag{16}$$

$$h^v_{n+1,m} = \frac{p_v}{n - m + 1} [(2n + 1) \sin \theta_v h^v_{n,m} - (n + m) p_v h^v_{n-1,m}] \tag{17}$$

To advance in m, use

$$f^v_{n+1,n+1} = (2n + 1) p_v [\cos \theta_v \cos \lambda_v f^v_{n,n} - \cos \theta_v \sin \lambda_v h^v_{n,n}] \tag{18}$$

$$h^v_{n+1,n+1} = (2n + 1) p_v [\cos \theta_v \cos \lambda_v h^v_{n,n} + \cos \theta_v \sin \lambda_v f^v_{n,n}] \tag{19}$$

Start the recurrences from

$$f^v_{0,0} = \frac{1}{\varrho_v} \tag{20}$$

$$f^v_{1,0} = R \frac{\sin \theta_v}{\varrho_v^2} \tag{21}$$

$$h^v_{0,0} = h^v_{1,0} = 0 \tag{22}$$

and terminate the procedure at $n = NMAX$.

Store the resulting ${}^\alpha F_{nm}$, ${}^\beta F_{nm}$, ${}^\alpha H_{nm}$ and ${}^\beta H_{nm}$. They will be programmed as constant parameters into the computer routine for the perturbing acceleration and are expected to serve for all subsequent computer runs of that routine, until updated. Note that the ${}^\alpha F_{nm}$, ${}^\beta F_{nm}$, ${}^\alpha H_{nm}$ and ${}^\beta H_{nm}$ are dimensionless quantities.

CALCULATION OF THE ASTRONOMICAL ARGUMENTS FOR THE VARIOUS TIDE COMPONENTS

Evaluate now, using *double precision* throughout,

$$\delta(n,m) = \begin{cases} 1 & \text{if } n = m \\ 0 & \text{if } n \neq m \end{cases} \tag{101}$$

$$\begin{aligned} N_\Sigma = {} & n\,\delta(J, 1975) + (365 + n)\,\delta(J, 1976) \\ & + (\ 731 + n)\,\delta(J, 1977) + (1096 + n)\,\delta(J, 1978) \\ & + (1461 + n)\,\delta(J, 1979) + (1826 + n)\,\delta(J, 1980) \\ & + (2192 + n)\,\delta(J, 1981) + (2557 + n)\,\delta(J, 1982) \\ & + (2922 + n)\,\delta(J, 1983) + (3287 + n)\,\delta(J, 1984) \\ & + (3653 + n)\,\delta(J, 1985) + (4018 + n)\,\delta(J, 1986) \end{aligned} \tag{102}$$

$$\Delta T = (5.28E\text{-}04) + (3.56E\text{-}08)\ N_\Sigma \tag{103}$$

$$d_0 = 27392.5 + N_\Sigma + \Delta T \tag{104}$$

$$T_0 = \frac{d_0}{36525} \tag{105}$$

$$\begin{aligned} h_0 = {} & 279.69668 \\ & + 36000.7689304850\ T_0 \\ & + 0.000303\ T_0^2 \end{aligned} \tag{106}$$

$$\begin{aligned} s_0 = {} & 270.434358 \\ & + 481267.88314137\ T_0 \\ & - 0.001133\ T_0^2 \\ & + 0.0000019\ T_0^3 \end{aligned} \tag{107}$$

$$\begin{aligned} p_0 = {} & 334.329653 \\ & + 4069.0340329575\ T_0 \\ & - 0.010325\ T_0^2 \\ & - 0.000012\ T_0^3 \end{aligned} \tag{108}$$

For each tide mode, find now the Astronomical Argument χ from

Tide Mode	χ in deg =	
M_2	$2\,(h_0 - s_0)$	(109)
S_2	0	(110)
N_2	$2\,h_0 - 3\,s_0 + p_0$	(111)

Tide Mode	χ in deg =	
K_2	$2h_0$	(112)
K_1	$h_0 + 90$	(113)
O_1	$h_0 - 2s_0 - 90$	(114)
P_1	$h_0 - 90$	(115)
Q_1	$h_0 - 3s_0 + p_0 - 90$	(116)
M_f	$2s_0$	(117)
M_m	$s_0 - p_0$	(118)
S_{sa}	$2h_0$	(119)

Note that the day number "n," and thus χ, must be updated whenever the time argument t^* runs into the next day.

COMPUTER ROUTINE FOR THE PERTURBING ACCELERATION

For each tide mode, separately, use the set of time-independent tide potential coefficients and the astronomical tide argument to evaluate the following algorithm.

First, calculate the earth-fixed Cartesian satellite coordinates (y_1, y_2, y_3) from the corresponding inertial coordinates (x_1, x_2, x_3), the latter being associated with the time line at t^* seconds from epoch.

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = (ABCD)_{j,n,t^*} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{201}$$

Find

$$r = +\sqrt{x_1^2 + x_2^2 + x_3^2} \tag{202a}$$

or

$$r = +\sqrt{y_1^2 + y_2^2 + y_3^2} \tag{202b}$$

as convenient.

Calculate now the time-dependent expansion coefficients for the tide potential (note: $n, m \leq NMAX$) from

$$F_{nm} = {}^{\alpha}F_{nm} \cos(\sigma t^* + \chi) + {}^{\beta}F_{nm} \sin(\sigma t^* + \chi) \tag{203}$$

$$H_{nm} = {}^{\alpha}H_{nm} \cos(\sigma t^* + \chi) + {}^{\beta}H_{nm} \sin(\sigma t^* + \chi) \tag{204}$$

The argument $(\sigma t^* + \chi)$ should be evaluated in double precision. The double precision sin and cos functions should be applied. Then calculate F_{nm} and H_{nm} and subsequently revert to single precision.

Note also that F_{nm} and H_{nm} are linear functions of the two trigonometric terms. To evaluate these trigonometric terms, let t_i^* and t_{i+1}^* be the values of Universal time for which subsequent integration steps are to be performed. To save computer time, update the trigonometric time factors as follows:

$$\cos(\sigma t_{i+1}^* + \chi) = \cos[(\sigma t_i^* + \chi) + \sigma\Delta t] = \cos(\sigma t_i^* + \chi)\cos\sigma\Delta t - \sin(\sigma t_i^* + \chi)\sin\sigma\Delta t \tag{205}$$

$$\sin(\sigma t_{i+1}^* + \chi) = \sin[(\sigma t_i^* + \chi) + \sigma\Delta t] = \sin(\sigma t_i^* + \chi)\cos\sigma\Delta t + \cos(\sigma t_i^* + \chi)\sin\sigma\Delta t \tag{206}$$

$$\Delta t = t_{i+1}^* - t_i^* \tag{207}$$

Now calculate the Cartesian components of the perturbing acceleration in the earth-fixed frame:

$$\frac{\partial \phi}{\partial y_1} = \sum_{n=0}^{NMAX} \sum_{m=0}^{n} \left(F_{nm} \frac{\partial U_{nm}}{\partial y_1} + H_{nm} \frac{\partial V_{nm}}{\partial y_1} \right) \tag{208}$$

$$\frac{\partial \phi}{\partial y_2} = \sum_{n=0}^{NMAX} \sum_{m=0}^{n} \left(F_{nm} \frac{\partial U_{nm}}{\partial y_2} + H_{nm} \frac{\partial V_{nm}}{\partial y_2} \right) \tag{209}$$

$$\frac{\partial \phi}{\partial y_3} = \sum_{n=0}^{NMAX} \sum_{m=0}^{n} \left(F_{nm} \frac{\partial U_{nm}}{\partial y_3} + H_{nm} \frac{\partial V_{nm}}{\partial y_3} \right) \tag{210}$$

where

$$\frac{\partial U_{nm}}{\partial y_1} = \frac{1}{R} \left(\frac{1}{2} A_{mn} U_{n+1,m-l} - \frac{1}{2} U_{n+1,m+l} \right) \tag{211}$$

$$\frac{\partial U_{nm}}{\partial y_2} = \frac{1}{R} \left(-\frac{1}{2} A_{mn} V_{n+1,m-l} - \frac{1}{2} V_{n+1,m+l} \right) \tag{212}$$

$$\frac{\partial U_{nm}}{\partial y_3} = \frac{-1}{R} (n - m + 1) U_{n+1,m} \tag{213}$$

$$\frac{\partial V_{nm}}{\partial y_1} = \frac{1}{R} \left(\frac{1}{2} A_{mn} V_{n+1,m-l} - \frac{1}{2} V_{n+1,m+l} \right) \tag{214}$$

$$\frac{\partial V_{nm}}{\partial y_2} = \frac{1}{R} \left(\frac{1}{2} A_{mn} U_{n+1,m-l} + \frac{1}{2} U_{n+1,m+l} \right) \tag{215}$$

$$\frac{\partial V_{nm}}{\partial y_3} = \frac{-1}{R} (n - m + 1) V_{n+1,m} \tag{216}$$

and

$$A_{mn} = (n - m + 1)(n - m + 2) \tag{217}$$

$$U_{n,-1} = - \frac{(n - 1)!}{(n + 1)!} U_{n1} = \frac{U_{n1}}{n(n + 1)} \tag{218}$$

$$V_{n,-1} = - \frac{(n - 1)!}{(n + 1)!} V_{n1} = - \frac{V_{n1}}{n(n + 1)} \tag{219}$$

Obtain the values of the individual eigenfunctions from the following recursive relationships:

$$p = \frac{R}{r} \tag{220}$$

$$\sin \psi = \frac{y_3}{r} \tag{221}$$

$$V_{n0} = 0 \text{ for all values of } n \tag{222}$$

$$U_{nm} = V_{nm} = 0 \text{ for all } n < m \tag{223}$$

To advance in n, evaluate

$$U_{n+1,m} = \frac{p}{n - m + 1} [(2n + 1) \sin \psi \, U_{nm} - (n + m) p \, U_{n-1,m}] \tag{224}$$

$$V_{n+1,m} = \frac{p}{n - m + 1} [(2n + 1) \sin \psi \, V_{nm} - (n + m) p \, V_{n-1,m}] \tag{225}$$

To advance in m, use

$$U_{n+1,n+1} = (2n + 1) p \left(\frac{y_1}{r} U_{nn} - \frac{y_2}{r} V_{nn} \right) \tag{226}$$

$$V_{n+1,n+1} = (2n + 1) p \left(\frac{y_1}{r} V_{nn} + \frac{y_2}{r} U_{nn} \right) \tag{227}$$

Start from

$$U_{0,0} = \frac{\mu_E}{r} \tag{228}$$

$$U_{1,0} = \frac{Ry_3}{r^3}\mu_E \tag{229}$$

$$V_{0,0} = V_{1,0} = 0 \tag{230}$$

The Cartesian components of the perturbing acceleration, in the earth-fixed reference frame, are now

$$T_{y_1} = \frac{\partial \phi}{\partial y_1} \tag{231}$$

$$T_{y_2} = \frac{\partial \phi}{\partial y_2} \tag{232}$$

$$T_{y_3} = \frac{\partial \phi}{\partial y_3} \tag{233}$$

Finally, rotate the perturbing acceleration vector back into the inertial coordinate system:

$$\begin{pmatrix} T_{x_1} \\ T_{x_2} \\ T_{y_3} \end{pmatrix} = (ABCD)^T_{J,n,\ell}\cdot \begin{pmatrix} T_{y_1} \\ T_{y_2} \\ T_{y_3} \end{pmatrix} \tag{234}$$

where $(ABCD)^T$ is the transpose of $(ABCD)$.

COMPUTER PROGRAM OUTPUT

Add up the x_1 components, T_{x1}, of the perturbing accelerations resulting from the eleven tide modes

$$T_{x1} = (T_{x1})_{M2} + (T_{x1})_{S2} + \cdots\cdots + (T_{x1})_{Ssa} \tag{301}$$

and also compute for the x_2 and x_3 components

$$T_{x2} = (T_{x2})_{M2} + \cdots\cdots\cdots + (T_{x2})_{Ssa} \tag{302}$$

$$T_{x3} = (T_{x3})_{M2} + \cdots\cdots\cdots + (T_{x3})_{Ssa} \tag{303}$$

These are the Cartesian components, in inertial space, of the perturbing acceleration caused by the combined action of all ocean tide modes considered plus the effect on the solid earth tide of tidal ocean loading (which, as already mentioned, had not been included in the formulation of the perturbing acceleration produced by the solid earth tide in the previous[4,5] tide routines).

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APPENDIX A

NOTES ON AUGMENTATION OF SOLID EARTH TIDE ALGORITHM BY OCEAN LOADING

NOTES ON AUGMENTATION OF SOLID EARTH TIDE ALGORITHM BY OCEAN LOADING

The outstanding structural difference between the above documented algorithms for the expansion coefficients of the tide potential and perturbing acceleration and the corresponding formulae and computer routines in References 3 and 5 is that the former account for eleven tide modes (including the M_2 tide) while the latter are restricted to the semidiurnal tide. Compared with it, the modification that now introduces the influence of hydrostatic ocean tide loading on the solid earth tide appears rather minor. In fact, it consists of just a slight alteration of the equations for the quantities α_{ij} and β_{ij} (Equations 01 and 02 in the main part of this report). The following is the theoretical background of this alteration*).

For each tide mode, the tidal elevation of the sea surface is

$$\begin{aligned}\xi_\nu &= \zeta_\nu \cos[(\sigma t^* + \chi) - \delta_\nu] \\ &= \zeta_\nu \cos\delta_\nu \cos(\sigma t^* + \chi) \\ &\quad + \zeta_\nu \sin\delta_\nu \sin(\sigma t^* + \chi)\end{aligned} \tag{401}$$

where the astronomical tide argument, χ, is a linear function of h_0, s_0 and p_0

$$\chi = fct\,[h_0(T_0), s_0(T_0), p_0(T_0)] \tag{402}$$

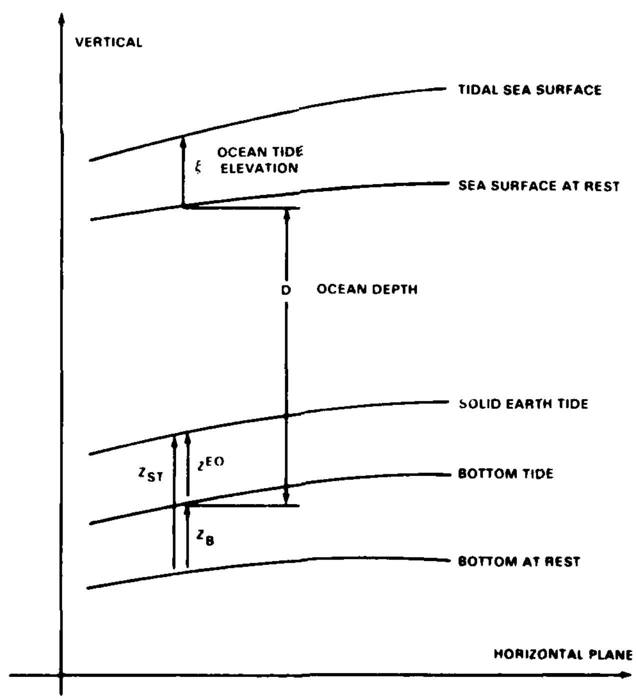

Figure 4. Geometrical Relationship Between Ocean Tide and Sea Bottom Tide.

* Note that except for ξ_ν definitions of the quantities occurring are unnecessary because they may be readily found in the main part of this report as well as in References 3 and 5. As before, units are *km* for length, *kg* for mass and *sec* for time except for the tidal amplitude on area element, ζ_{ij}, specified in *cm*.

T_0 is the fractional time in centuries at the beginning of the (mean solar) day UT. The point mass m_ν representing that portion of the tide bulge which is located within the particular surface area element ΔS_ν (Reference 3, Page 3) is

$$m_\nu = \frac{1}{G}\left[\alpha_\nu \cos(\sigma t^* + \chi) + \beta_\nu \sin(\sigma t^* + \chi)\right] \tag{403}$$

$$\alpha_\nu = \varrho G \, \Delta S_\nu \, 10^{-3} \, \zeta_\nu \cos\delta_\nu \tag{404}$$

$$\beta_\nu = \varrho G \, \Delta S_\nu \, 10^{-3} \, \zeta_\nu \sin\delta_\nu \tag{405}$$

Defining a "gravitational constant" μ_ν for each point mass one can also say

$$\mu_\nu = Gm_\nu = \alpha_\nu \cos(\sigma t^* + \chi) + \beta_\nu \sin(\sigma t^* + \chi) \tag{406}$$

If one now recalls[3,5] that the perturbing potential aloft caused by the ocean tide may be expressed as

$$\phi = \sum_{n=0}^{NMAX} \sum_{m=0}^{n} \left[F_{nm} U_{nm} + H_{nm} V_{nm}\right] \tag{407}$$

$$F_{nm} = \sum_{\nu=1}^{N} F_{nm}^{\nu} \tag{408}$$

$$H_{nm} = \sum_{\nu=1}^{N} H_{nm}^{\nu} \tag{409}$$

$$F_{nm}^{\nu} = (2 - \delta_m^0)\frac{\mu_\nu}{\mu}\,\frac{(n-m)!}{(n+m)!}\,\frac{\varrho_\nu^n}{R^n}\,P_n^m(\sin\theta_\nu)\cos m\lambda_\nu \tag{410}$$

$$H_{nm}^{\nu} = \frac{\mu_\nu}{\mu}\,\frac{(n-m)!}{(n+m)!}\,\frac{\varrho_\nu^n}{R^n}\,P_n^m(\sin\theta_\nu)\cos m\lambda_\nu \tag{411}$$

$$U_{nm} = \frac{\mu R^n}{r^{n+1}}\,P_n^m(\sin\theta)\cos m\lambda \tag{412}$$

$$V_{nm} = \frac{\mu R^n}{r^{n+1}}\,P_n^m(\sin\theta)\cos m\lambda \tag{413}$$

it becomes evident that it is the quantities μ_ν which introduce time into the expansion coefficients F_{nm} and H_{nm} of the potential.

To realize how the effect of ocean tide loading may be formulated, consider Figure 4. This illustrates the interaction, by hydrostatic loading, of the ocean tide with the solid earth tide. The present approach is based on the fortunate circumstance that, as evidenced by Reference 6 (see

especially Equation 8), Reference 10 and Reference 8 (see Equation 3), the dip z^{EO} of the solid earth (ocean floor) in response to the oceanic tidal load is proportional to the ocean tide elevation ξ:

$$Z_{ST} - Z_B = Z^{EO} \approx + 0.0667\,\xi \tag{414}$$

One must further keep in mind that the perturbing acceleration based on the solid earth tide, as previously formulated[4,5], contains the effect of that portion of the oceanic crust that would in the absence of hydrostatic ocean tide loading be located between the surfaces marked "Solid Earth Tide" and "Bottom Tide" in Figure 4. Because the sea floor is now depressed by Z^{EO}, we must nullify the just mentioned part of the solid tide. This may conveniently be done by asserting that the missing mass is a negative mass layer whose gravity acting on the satellite will offset the unwanted masses present in the existing version of the solid earth tide algorithm.

Making use of the fact that the ocean depth is a small fraction of the earth's radius, the perturbing acceleration due to this negative mass layer (density: ϱ^*, located on the ocean surface) may now be derived in exactly the same manner in which the perturbation due to the sea tide was developed in Reference 3. There will result a perturbing potential identical to Equations 403 through 413 above, except that the water density ϱ is replaced by - 0.0667 ϱ^* (see "Input Data" in the main part of this report). The combined gravitational action of the ocean tide and the just defined "missing mass layer" is then reflected in the main part of this report, the only difference between it and the case of a pure ocean tide consisting of the term - 0.0667 ϱ^* in Equations 01, 02, 415 and 416,

$$\alpha_\nu = (\varrho - 0.0667\,\varrho^*)\,G\,\Delta S_\nu\,10^{-3}\,\zeta_\nu \cos\delta_\nu \tag{415}$$

$$\beta_\nu = (\varrho - 0.0667\,\varrho^*)\,G\,\Delta S_\nu\,10^{-3}\,\zeta_\nu \sin\delta_\nu \tag{416}$$

Otherwise, the ocean tide algorithm remains undisturbed.

APPENDIX B

TRIAL DATA FOR COMPUTER PROGRAM CHECKOUT

TRIAL DATA FOR COMPUTER PROGRAM CHECKOUT

The following trial calculations are intended to facilitate computer program checkout. They were done on an electronic calculator. To be feasible, they were restricted to the lunar semidiurnal (M_2) tide mode. Also, for simplicity, the density of the ocean floor rock, ϱ^*, was assumed zero.

INPUT DATA

$$NP = 9$$

Quantities Associated with the Surface Area Elements

ij	v	ζ m	δ deg	θ rad	λ rad	ϱ km	ΔS_v km^2	α_v $km^3\ sec^{-2}$	β_v $km^3\ sec^{-2}$
1,1	1	10	25	1.562069681	8.7266463 E-03	6356.800824	108.1411251	6.5403462E-08	3.0498135E-08
2,1	2	10	25	1.562069681	2.61799388E-02	6356.800824	108.1411251	6.5403462E-08	3.0498135E-08
3,1	3	10	25	1.562069681	4.36332313E-02	6356.800824	108.1411251	6.5403462E-08	3.0498135E-08
1,2	4	10	25	1.544616388	8.7266463 E-03	6356.813825	432.4766612	2.6156072E-07	1.2196777E-07
2,2	5	20	30	1.544616388	2.61799388E-02	6356.813825	432.4766612	4.9987043E-07	2.8860033E-07
3,2	6	20	30	1.544616388	4.36332312E-02	6356.813825	432.4766612	4.9987043E-07	2.8860033E-07
1,3	7	10	25	1.527163095	8.7266463 E-03	6356.839812	648.550316	3.9224149E-07	1.8290521E-07
2,3	8	20	30	1.527163095	2.61799388E-02	6356.839812	648.550316	7.496153 E-07	4.327906 E-07
3,3	9	20	30	1.527163095	4.36332313E-02	6356.839812	648.550316	7.496153 E-07	4.327906 E-07

$$R = 6378.145\ km$$

$$\varepsilon^2 = 0.00669342$$

$$\mu_E = 398601\ km^3\ sec^{-2}$$

$$G = 6.6732\text{E-}20\ km^3\ kg^{-1}\ sec^{-2}$$

$$\varrho = 1.E+12\ kg\ km^{-3}$$

$$\varrho^* = 0$$

$$\sigma = \frac{180}{\pi}\ 1.40519\text{E-}04\ deg\ sec^{-1} \quad \text{(evaluate in double precision)}$$

$$\frac{1}{\pi} = .3183098861837906715 38$$

$$J = 1977$$

$$n = 202$$

$$t^* = 50000 \, sec$$

$$x_1 = +\ 3151.52923 \, km$$

$$x_2 = +\ 5458.60875 \, km$$

$$x_3 = +\ 3639.07250 \, km$$

$$(ABCD)_{J,n,t^*} = \begin{pmatrix} \mathrm{A11} & \mathrm{A12} & \mathrm{A13} \\ \mathrm{A21} & \mathrm{A22} & \mathrm{A23} \\ \mathrm{A31} & \mathrm{A32} & \mathrm{A33} \end{pmatrix}$$

$$\mathrm{A11} = -\ .8405285753$$

$$\mathrm{A12} = +\ .5417623775$$

$$\mathrm{A13} = +\ .2289080162 \ \mathrm{E\text{-}02}$$

$$\mathrm{A21} = -\ .5417605355$$

$$\mathrm{A22} = -\ .8405316908$$

$$\mathrm{A23} = +\ .1413662999 \ \mathrm{E\text{-}02}$$

$$\mathrm{A31} = +\ .2689913850 \ \mathrm{E\text{-}02}$$

$$\mathrm{A32} = -\ .5190827376 \ \mathrm{E\text{-}04}$$

$$\mathrm{A33} = +\ .9999963803$$

$$NMAX = 4$$

COMPUTATION OF TIME-INDEPENDENT POTENTIAL COEFFICIENTS

Values for the f^{ν}_{nm} and h^{ν}_{nm}

ν	$f^{\nu}_{0,0}$ km^{-1}	$h^{\nu}_{0,0}$ km^{-1}	$f^{\nu}_{1,0}$ km^{-1}	$h^{\nu}_{1,0}$ km^{-1}
1	1.5731184E − 04	0	1.5783403E − 04	0
2	1.5731184E − 04	0	1.5783403E − 04	0
3	1.5731184E − 04	0	1.5783403E − 04	0
4	1.5731151E − 04	0	1.5778531E − 04	0
5	1.5731151E − 04	0	1.5778531E − 04	0
6	1.5731151E − 04	0	1.5778531E − 04	0
7	1.5731087E − 04	0	1.5768788E − 04	0
8	1.5731087E − 04	0	1.5768788E − 04	0
9	1.5731087E − 04	0	1.5768788E − 04	0

ν	$f^{\nu}_{1,1}$ km^{-1}	$h^{\nu}_{1,1}$ km^{-1}	$f^{\nu}_{2,0}$ km^{-1}	$h^{\nu}_{2,0}$ km^{-1}
1	1.3773442E − 06	1.2019901E − 08	1.5835193E − 04	0
2	1.3769246E − 06	3.6056041E − 08	1.5835193E − 04	0
3	1.3760857E − 06	6.0081198E − 08	1.5835193E − 04	0
4	4.1315963E − 06	3.6055895E − 08	1.5820627E − 04	0
5	4.1303378E − 06	1.0815670E − 07	1.5820627E − 04	0
6	4.1278211E − 06	1.8022456E − 07	1.5820627E − 04	0
7	6.8845393E − 06	6.0080464E − 08	1.5791513E − 04	0
8	6.8824422E − 06	1.8022309E − 07	1.5791513E − 04	0
9	6.8782468E − 06	3.0031082E − 07	1.5791513E − 04	0

ν	$f^{\nu}_{2,1}$ km^{-1}	$h^{\nu}_{2,1}$ km^{-1}	$f^{\nu}_{2,2}$ km^{-1}	$h^{\nu}_{2,2}$ km^{-1}
1	4.1457488E − 06	3.6179402E − 08	3.6175267E − 08	6.3144163E − 10
2	4.1444859E − 06	1.0852718E − 07	3.6131193E − 08	1.8935556E − 09
3	4.1419607E − 06	1.8084191E − 07	3.6043099E − 08	3.1533625E − 09
4	1.2432120E − 05	1.0849347E − 07	3.2550933E − 07	5.6817864E − 09
5	1.2428333E − 05	3.2544735E − 07	3.2511274E − 07	1.7038437E − 08
6	1.2420760E − 05	5.4230210E − 07	3.2432006E − 07	2.8374329E − 08
7	2.0703116E − 05	1.8067335E − 07	9.0381431E − 07	1.5776137E − 08
8	2.0696809E − 05	5.4196503E − 07	9.0271315E − 07	4.7309191E − 08
9	2.0684198E − 05	9.0309161E − 07	9.0051217E − 07	7.8784606E − 08

Values for the f^{ν}_{nm} and h^{ν}_{nm} (Continued)

ν	$f^{\nu}_{3,0}$ km^{-1}	$h^{\nu}_{3,0}$ km^{-1}	$f^{\nu}_{3,1}$ km^{-1}	$h^{\nu}_{3,1}$ km^{-1}
1	1.5886548E − 04	0	8.3188628E − 06	7.2597615E − 08
2	1.5886548E − 04	0	8.3163287E − 06	2.1777073E − 07
3	1.5886548E − 04	0	8.3112614E − 06	3.6287751E − 07
4	1.5857391E − 04	0	2.4934851E − 05	2.1760315E − 07
5	1.5857391E − 04	0	2.4927255E − 05	6.5274316E − 07
6	1.5857391E − 04	0	2.4912067E − 05	1.0876843E − 06
7	1.5799154E − 04	0	4.1485684E − 05	3.6204008E − 07
8	1.5799154E − 04	0	4.1473047E − 05	1.0860100E − 06
9	1.5799154E − 04	0	4.1447777E − 05	1.8096490E − 06

ν	$f^{\nu}_{3,2}$ km^{-1}	$h^{\nu}_{3,2}$ km^{-1}	$f^{\nu}_{3,3}$ km^{-1}	$h^{\nu}_{3,3}$ km^{-1}
1	1.8147675E − 07	3.1676885E − 09	1.5834220E − 09	4.1463365E − 11
2	1.8125565E − 07	9.4992060E − 09	1.579082 E − 09	1.2427645E − 10
3	1.8081372E − 07	1.5819151E − 08	1.5704138E − 09	2.0674889E − 10
4	1.6324485E − 06	2.8494495E − 08	4.2739029E − 08	1.1191609E − 09
5	1.6304596E − 06	8.5448767E − 08	4.2621885E − 08	3.3544151E − 09
6	1.6264843E − 06	1.4229893E − 07	4.2387916E − 08	5.58047 E − 09
7	4.5299018E − 06	7.9069730E − 08	1.9774213E − 07	5.1780599E − 09
8	4.5243828E − 06	2.3711286E − 07	1.9720013E − 07	1.5519987E − 08
9	4.5133516E − 06	3.9486710E − 07	1.9611762E − 07	2.5819375E − 08

ν	$f^{\nu}_{4,3}$ km^{-1}	$h^{\nu}_{4,3}$ km^{-1}
1	1.1120747E − 08	2.9120701E − 10
2	1.1090266E − 08	8.7282284E − 10
3	1.1029387E − 08	1.4520463E − 09
4	3.0007426E − 07	7.8577211E − 09
5	2.9925178E − 07	2.3551626E − 08
6	2.9760907E − 07	3.9180977E − 08
7	1.3875122E − 06	3.6333286E − 08
8	1.3837091E − 06	1.0890027E − 07
9	1.3761134E − 06	1.8116877E − 07

Time-Independent Coefficients of the Tide Potential

n	m	$^{\alpha}F_{nm}$	$^{\beta}F_{nm}$	$^{\alpha}H_{nm}$	$^{\beta}H_{nm}$
0	0	8.4018456E − 12	4.6140106E − 12	0	0
1	0	8.3681641E − 12	4.5954868E − 12	0	0
1	1	2.9297877E − 13	1.6202500E − 13	4.3123569E − 15	2.4544968E − 15
2	0	8.3290386E − 12	4.5739464E − 12	0	0
2	1	2.9177584E − 13	1.6135948E − 13	4.2946458E − 15	2.4444118E − 15
2	2	2.7840515E − 15	1.5423222E − 15	8.2132887E − 17	4.683368 E − 17
3	0	8.2845357E − 12	4.5494263E − 12	0	0
3	1	2.9046632E − 13	1.6063487E − 13	4.2753632E − 15	2.4334303E − 15
3	2	2.7724443E − 15	1.5358913E − 15	8.1790436E − 17	4.6638388E − 17
3	3	1.8513081E − 17	1.0259185E − 17	8.2068535E − 19	4.6816230E − 19
4	3	1.8435105E − 17	1.0215973E − 17	8.1722863E − 19	4.6619036E − 19

COMPUTATION OF THE PERTURBING ACCELERATION

$$N_\Sigma = 731 + 202 = 933 \text{ exactly}$$

$$\Delta T = .000561214800000 \text{ exactly}$$

$$d_0 = 28325.500561214800000 \text{ exactly}$$

$$T_0 = .775509940074327173169062286105$$

$$h_0 = .28198651018139069055E + 05$$

$$s_0 = .37349846089214435790E + 06$$

$$\chi (M_2) = 2 (h_0 - s_0)$$

$$= - .69059961974801057768E + 06 \text{ deg}$$

$$(\sigma t^* + \chi) = - .69019706246594063695E + 06 \text{ deg}$$

$$= - .12046211227623637380E + 05 \text{ rad}$$

$$\cos (\sigma t^* + \chi) = + .223888627216$$

$$\sin (\sigma t^* + \chi) = - .974614735474$$

$$y_1 = + 316.64861 \; km$$

$$y_2 = - 6290.36338 \; km$$

$$y_3 = + 3647.25332 \; km$$

$$r = 7278.144995 \; km$$

$$p = .8763421179$$

$$\sin \psi = .5011240257$$

$$U_{00} = 54.76683966$$

$$U_{10} = 24.05119117$$

Time-Dependent Coefficients of the Tide Potential

n	m	F_{nm}	H_{nm}
0	0	− 2.615805043 E-12	0
1	0	− 2.605292379 E-12	0
1	1	− 9.231733790 E-14	− 1.426701083 E-15
2	0	− 2.593058543 E-12	0
2	1	− 9.193803464 E-14	− 1.420837407 E-15
2	2	− 8.798524745 E-16	− 2.725617532 E-17
3	0	− 2.579124585 E-12	0
3	1	− 9.152500570 E-14	− 1.414451830 E-15
3	2	− 8.761835447 E-16	− 2.714251175 E-17
3	3	− 5.853884584 E-18	− 2.725357598 E-19

Partial Derivatives of Eigenfunctions

n	m	$\frac{\partial}{\partial y_1} U_{nm}$	$\frac{\partial}{\partial y_1} V_{nm}$
0	0	− .0003273812925	0
1	0	− .0004313144646	0
1	1	+ .006556883732	+ .0007438816215
2	0	− .00009640471307	0
2	1	+ .008605596947	+ .001633400725
2	2	+ .004318488243	− .02968388935
3	0	+ .0003428662534	0
3	1	+ .001887360387	+ .001082405391
3	2	+ .01195047744	− .06493019830
3	3	− .1675467571	− .03196077254

n	m	$\frac{\partial}{\partial y_2} U_{um}$	$\frac{\partial}{\partial y_2} V$
0	0	+ .006503572822	0
1	0	+ .008568250824	0
1	1	+ .0007438816215	− .008183204595
2	0	+ .001915121866	0
2	1	+ .001633400725	− .023760040082
2	2	− .02585364562	− .004125678817
3	0	− .006811188356	0
3	1	+ .001082405391	− .01956061011
3	2	− .1057973284	− .01400767497
3	3	− .02763115097	+ .1246508161

n	m	$\frac{\partial}{\partial y_3} U_{nm}$	$\frac{\partial}{\partial y_3} V_{nm}$
0	0	− .003770875565	0
1	0	+ .001626320863	0
1	1	− .0004313144646	+ .008568250824
2	0	+ .007577401923	0
2	1	− .0001928094261	+ .003830243731
2	2	+ .03236599779	+ .003266801449
3	0	+ .005891083242	0
3	1	+ .00102859876	− .02043356501
3	2	+ .04289594100	− .004329621566
3	3	+ .02595815241	− .1707275267

$$T_{y_1} = -\ 2.391047870 \text{ E-16 } km\ sec^{-2}$$

$$T_{y_2} = -\ 2.686274153 \text{ E-14 } km\ sec^{-2}$$

$$T_{y_3} = -\ 2.930738682 \text{ E-14 } km\ sec^{-2}$$

$$T_{x_1} = +\ 1.467531330 \text{ E-14 } km\ sec^{-2}$$

$$T_{x_2} = +\ 2.245096888 \text{ E-14 } km\ sec^{-2}$$

$$T_{x_3} = -\ 2.934580293 \text{ E-14 } km\ sec^{-2}$$

Check:

$$T_x = [T_{x_1}^2 + T_{x_2}^2 + T_{x_3}^2]^{1/2} = 3.975659661 \text{ E-14 } km\ sec^{-2}$$

$$T_y = [T_{y_1}^2 + T_{y_2}^2 + T_{y_3}^2]^{1/2} = 3.975659664 \text{ E-14 } km\ sec^{-2}$$

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K10	
K104 (Schwiderski)	
K12	(40)
K12 (Groeger)	(40)
K11	
K13	
K14	(5)
X210	(6)

ILMED

8